Politecnico di Torino
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PhD in Computer and Control Engineering
XXIV cycle

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Fixed-Order Fixed-Structure Frequency Domain Control Design

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1. Introduction

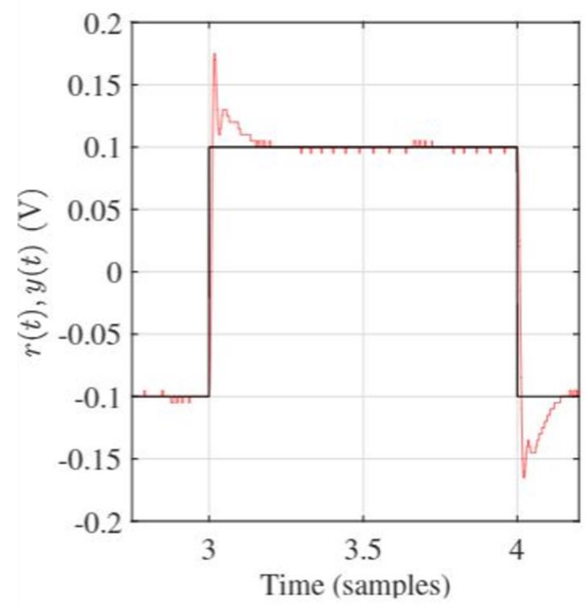
The development of a worst-case control design for a linear time invariant plants subjected to unknown parameter uncertainties and disturbances is a topic of great interest for the control community. This is motivated by the fact that the real systems are always uncertain to some extent. One of the popular method for robust control design is H∞ mixed-sensitivity framework. In this framework, H∞-norm is used to define constraints on both the sensitivity and complementary sensitivity function to ensure good performances and the robustness of the system to be controlled. Classical algorithms for H∞ control synthesis cannot take into account the order of the controller. However, in practical applications, the controller structure and/ order is a-priori fixed and cannot be modified. Thus, the design of fixed-order fixed structure (FOFS) remains an open topic.

2. Research goal

The aim of this PhD research activity is to investigate novel approaches to effectively solve the FOFS H∞ control design problem by properly exploiting recent results obtained in the area of polynomial optimization. The main difficulty in the design of FOFS H∞ control is that the constraints on the structure and/ or order of the controller makes the optimization problem nonconvex and difficult to solve. FOFS control design approaches based on local optimization methods typically trap in local optima and therefore could not guarantee to find feasible controller fulfilling robust stability and robust performance. In the proposed methods, suitable convex relaxation techniques have been exploited to efficiently solve the polynomial optimization problems arising from the formulation of the FOFS H∞ control synthesis.

3. H∞ mixed-sensitivity design of FOFS controllers through Putinar positivstellensatz

In this method we assume that the transfer function of the plant is known. We first define the set of controller parameters that achieve robust stability and nominal performances of the feedback control system. Then, we rewrite the controller design problem as the positivity test over a bounded domain. By exploiting Putinar positivstellensatz theorem, we formulate the H∞ mixed sensitivity controller design as the non-emptiness test of a convex set defined through a number of sum of squares (SOS) polynomial constraints. The problem to be solved is a convex semi-definite problem (SDP), whose solution can be found in polynomial time ([1] and [2]). This technique is applied to design a PI+Lead controller for magnetic levitation system [1].



4. FOFS H∞ mixed-sensitivity DDDC design

In this method, FOFS H∞ mixed-sensitivity direct data-driven controllers (DDDC) are designed using experimental data from the plant. Given the assigned controller structure and a set of frequency-domain input-output data, we first define a set of controller parameters that guarantee closed-loop stability and satisfy the H∞ norm of performance constraints. The problem is then reformulated as a polynomial optimization problem and solved by means of convex relaxation techniques. The major highlights of the proposed techniques are:

- Necessary and sufficient conditions for closed stability in a direct data-driven framework are presented.
- FOFS robust controllers are designed using a frequency domain bound for unstructured uncertainty. Because of the estimation of frequency domain weighting filter for the uncertainty, control system design is a 2-step process.
- FOFS robust controllers are designed assuming that the uncertainty is unknown but bounded and belongs to a given semi-algebraic set. This results in a semi-infinite polynomial optimization problem which is solved through exchange algorithm by applying moment relaxation. In this way, control system is designed in a single step.

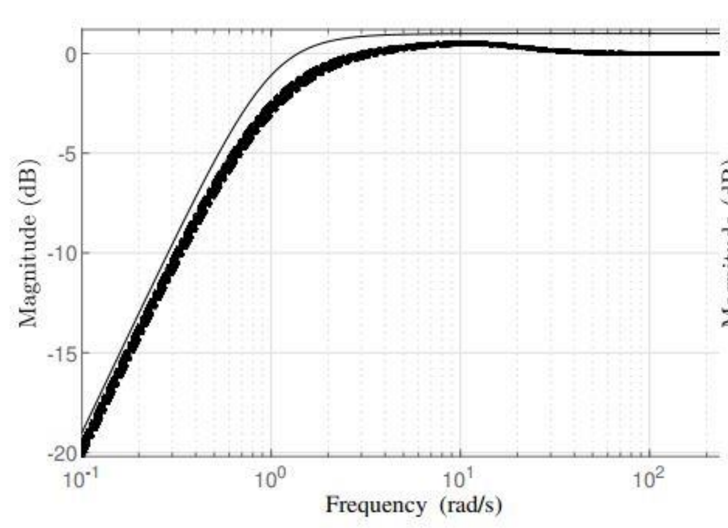


Figure: $|W_1^{-1}(s)|$ (solid) vs $|S_n(s)|$ (dotted)

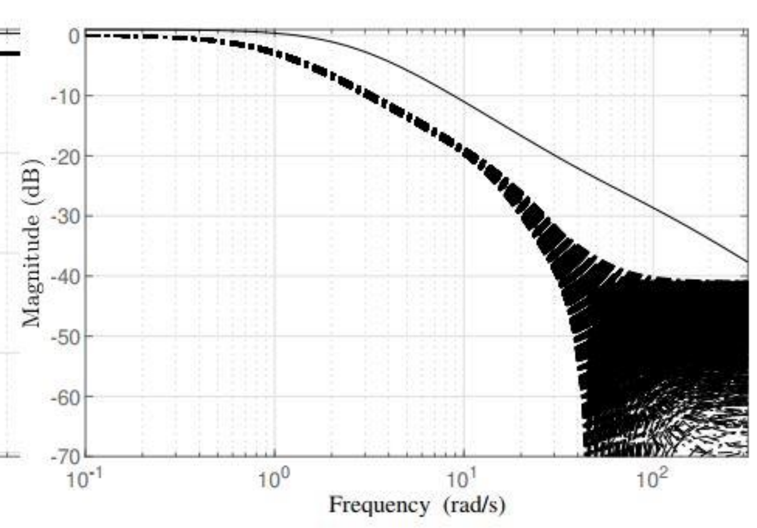


Figure: $|W_2^{-1}(s)|$ (solid) and $|T_n(s)|$ (dotted)

References

1. V. Razza, A. Salam, "A Unified Framework for the H∞ Mixed-Sensitivity Design of Fixed Structure Controllers through Putinar Positivstellensatz", In Machines, vol. 9 (8), pp-176,2021.
2. V. Cerone, S. Fosson, D. Regrutto, "Bode envelope bounds computation for linear time-invariant systems affected by semi-algebraic parametric uncertainty" In 16th IEEE conference on control & automation, Japan, pp-247-252, 2020.