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Knapsack Problems with Side Constraints

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1. Introduction

Everyday life aspects and directions are guided by decisions. Although passions and emotions play a fundamental role in taking small and big decisions, many professional contexts require a formalization of the decision-making process for solving complex problems. This is the goal of Combinatorial Optimization (CO), which provides methods and tools for solving optimization problems. My research activities were supported by a fellowship from TIM Joint Open Lab SWARM (Turin, Italy) and focused on a specific class of allocation resource problems: the **Knapsack Problems with Side Constraints**. These are classical problems in CO where a set of items with given profits and weights is available. The aim is to select a subset of the items in order to maximize the total profit without exceeding a known knapsack capacity. In the classical 0-1 Knapsack Problem (KP), each item can be picked at most once. KP can be formally stated as follows: a capacity value c and a set of n items j with weight w_j and profit p_j are given. The problem can be formulated as the following Integer Linear Programming (ILP) model

$$\text{Maximize} \quad \sum_{j=1}^{n} p_j x_j$$
$$\text{Subject to} \quad \sum_{j=1}^{n} w_j x_j \le c$$
$$x_j \in \{0,1\} \quad j = 1, \dots, n$$

where binary variables $x_j = 1$ if item j is placed in the knapsack, $x_j = 0$ otherwise. The capacity constraint represents the presence of limited resources for the choice of the items. I investigated three generalizations of KP (see Figure 1) involving side constraints beyond the capacity bound. These problems belong to the class of problems which are hard to solve (NP-hard problems) and thus pose challenging research topics.

2. Objectives and methods

The aim of my research was to provide scientific contributions from both a practical and theoretical perspective. On the one hand, we devised effective algorithms either for direct applications or when the problems considered arise as sub-problems in broader contexts. On the other hand, we gave insights into the structure and the properties of the problems and derived a series of theoretical results for some of them. In particular, I studied and leveraged methods and techniques in CO, such as integer programming and dynamic programming, for developing **exact solution approaches**. As appealing alternatives, I also investigated the existence of **approximation algorithms** whose solutions are sub-optimal but guaranteed to be sufficiently close to an optimal solution.

3. Problems and results

The 0-1 Knapsack Problem with setups (KPS)

Items belong to families (or classes) and can be selected only if the corresponding family is activated. The selection of a family involves setup costs and resource consumptions thus affecting both the profits and the capacity constraint.

Applications: resource allocation problems involving classes of elements such as in make-to-order production context and cargo loading. A relevant application comes from the smart-home paradigm (see Project FLEXMETER funded by the European Commission under H2020: http://flexmeter.polito.it). Here, energy providers are requested to manage peak demands to avoid blackouts while satisfying an aggregated demand curve. In this context, it may be required to shut down several home appliances whenever a Demand Response event for overall exceeding energy consumption is identified. This corresponds to select the best appliances to be shut down, by taking into account their relevance and their energy consumption, while also minimizing the houses involved in this shut down (see Figure 2). The problem can be seen as a KPS where the families of items are the houses that we do not want to shut down and the items are their appliances.

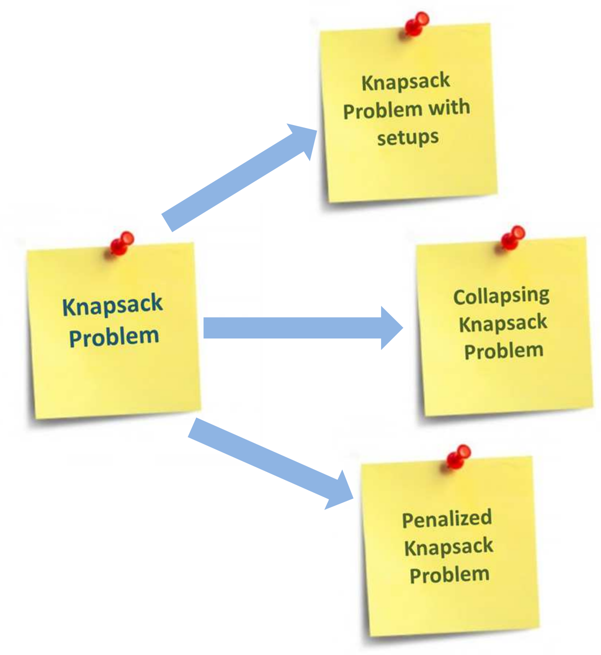

Figure 1: Knapsack Problems with Side Constraints

Scientific contributions:

- A new exact approach which effectively handles the structure of the ILP formulation of KPS. The algorithm exploits the partitioning of the variables set into two levels and requires the solution of several ILP models that are easy to solve in practice.

- A new effective dynamic programming algorithm which avoids the use of an ILP solver. The algorithm relies on a proper exploration of sub-problems which are then combined to reach an optimal solution.

- A general inapproximability result and approximation algorithms for special cases of KPS arising from certain additional, but plausible, restrictions on the input data.

Results:

- The solution approaches very favorably compare to the algorithms in literature and to the commercial solver CPLEX 12.5 applied to the ILP formulation.

- Optimal solutions within limited CPU time for instances with up to 100,000 items and 200 families (instances in the literature limited to 10,000 items and 30 families).

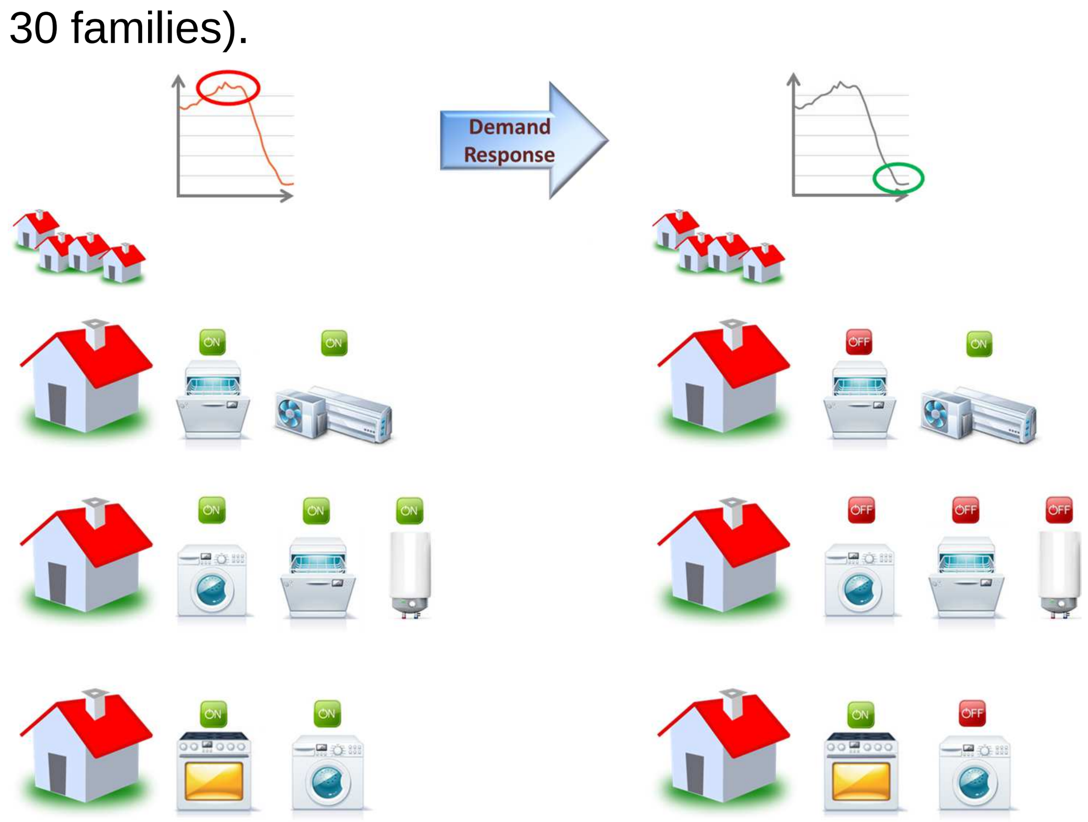

Figure 2: KPS application within the smart-home paradigm

The 0-1 Collapsing Knapsack Problem (CKP)

The capacity of the knapsack is not a scalar but a non-increasing function inversely related to the number of items placed inside the knapsack. This function represents situations where an overhead of the resources is produced by the number of items included in a solution.

Applications: problems with resource overheads, for example in time-sharing computer systems, satellite communication, transportation of fragile products.

Scientific contributions:

- A novel ILP formulation of CKP and an effective reduction procedure for restricting the solution space.

- An exact approach also extended to multidimensional variants of CKP involving up to 5 capacity constraints. The approach relies on the new ILP formulation of CKP and shares the general algorithmic idea for KPS of inducing problems tractable in practice through an effective exploration of the solution space of first level variables.

Results:

- By exploiting the potentials of the modern ILP solvers, the novel model provides already a significant contribution in solving to optimality CKP instances with up to 30,000 items (instances in the literature limited to 1,000 items).

- The exact approach is capable of effectively solving to optimality instances with up to 100,000 items.

- The proposed approach performs well also for the multidimensional variants of CKP, in particular for the variant involving two capacity constraints.

The 0-1 Penalized Knapsack Problem (PKP)

Each item has a profit, a weight and a penalty. The problem calls for maximizing the sum of the profits minus the greatest penalty value of the items selected in a solution.

Applications: PKP arises as sub-problem in approaches, such as branch-and-price algorithms, for solving complex packing problems in industrial applications.

Scientific contributions:

- A dynamic programming-based exact approach which effectively leverages an algorithmic framework devised for KP.

- A theoretical study of some properties of the problem and a general inapproximability result.

Results:

- The proposed approach turns out to be very effective. It outperforms the solver CPLEX 12.5 and a previous exact algorithm in the literature.

- All hard instances with up to 10,000 items solved to optimality with a CPU time limit of 100 seconds.

4. Future developments

- Future research will be devoted to extending our procedures to other variants of KP or to other optimization problems involving two sets of variables.

- A multi-period variant of KP, namely the *Incremental Knapsack Problem*, is currently investigated.

5. References

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2. F. Della Croce, F. Salassa, R. Scatamacchia. An exact approach for the 0-1 knapsack problem with setups. *Computers & Operations Research*, 80:61-67, 2017.
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